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QUIPS AND TIPS ON CUISENAIRE.
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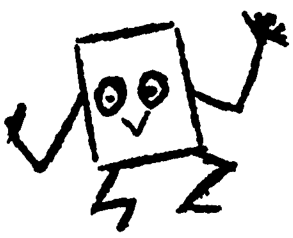
DESCRIPTORS- *MATHEMATICAL CONCEPTS, *MATHEMATICS MATERIALS, *ELEMENTARY GRADES, MATHEMATICS CURRICULUM, NUMBER CONCEPTS, MATHEMATICAL ENRICHMENT, *FUNDAMENTAL CONCEPTS, TEACHING GUIDES, CUISENAIRE MATERIALS, UNIVERSITY CITY

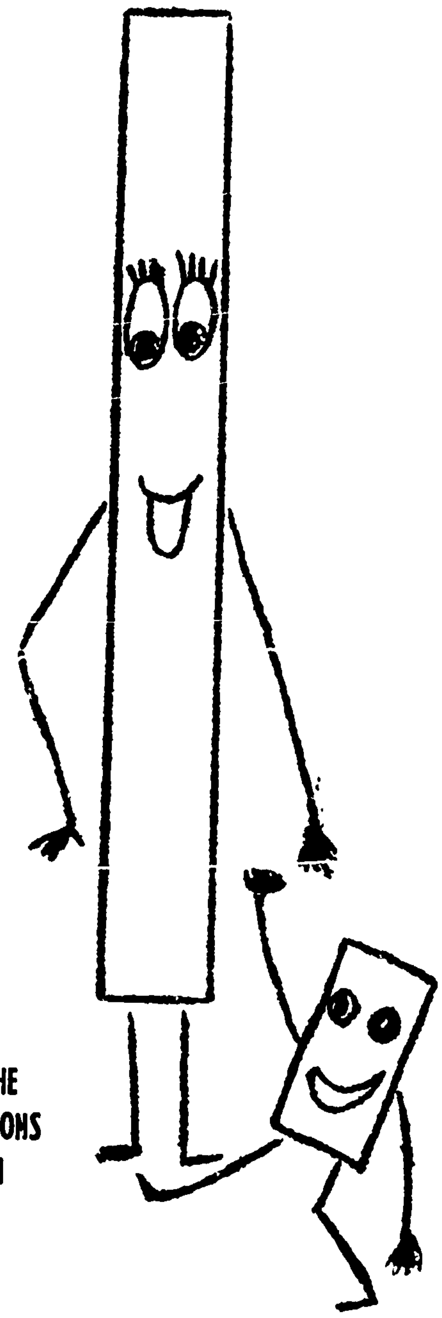
THIS DOCUMENT IS A TEACHING GUIDE TO AID IN THE USE OF CUISENAIRE MATERIALS TO PRESENT FUNDAMENTAL MATHEMATICAL CONCEPTS TO ELEMENTARY SCHOOL CHILDREN. CONCEPTS OF AND EXERCISES IN COUNTING, NUMBERING, ADDING, DIVIDING, SUBTRACTING, AND MULTIPLYING ARE EMPHASIZED. IN ADDITION, CONCEPTS OF FRACTIONS, DISTRIBUTIVE PROPERTIES OF NUMBERS, RECTANGULAR NUMBERS, TRIANGULAR NUMBERS, AND PERMUTATIONS ARE DISCUSSED. A FEW CLASSROOM ACTIVITIES (GAMES, DRAWING ACTIVITIES, AND RHYTHM ACTIVITIES ARE PRESENTED. A LIST OF DEFINITIONS OF SYMBOLS AND TERMS USED IN THE DOCUMENT AND A BIBLIOGRAPHY OF RELATED MATERIALS ARE INCLUDED. (FS)

"A rod is a piece of wood..."

"A rod is a pretty color..."

"A rod is a number..."





quips and tips on

Cuisenaire

ERRATA

Page 1 - Line 9:	The punctuation mark after the work "possible" should be a period.
Page 7 -Line 20:	"red" should be "rod".
Page 10:	The "+" is omitted from the first table.
Page 11 - Line 1:	"red" should be "rod".
Page 13:	Line 5 should be as follows: As you move along in the development of concepts of fractions you meet the problem of teaching how to multiply a fraction by a fraction.
Page 14 - Line 5:	The minus sign should be an equal sign.
Page 15 - Line 3:	The minus sign should be an equal sign.
Page 15:	2 g + p should be 2((g + p) under the first diagram.

quips and tips on

CUISENAIRE

For the Elementary Schools in the
School District of University City, Missouri

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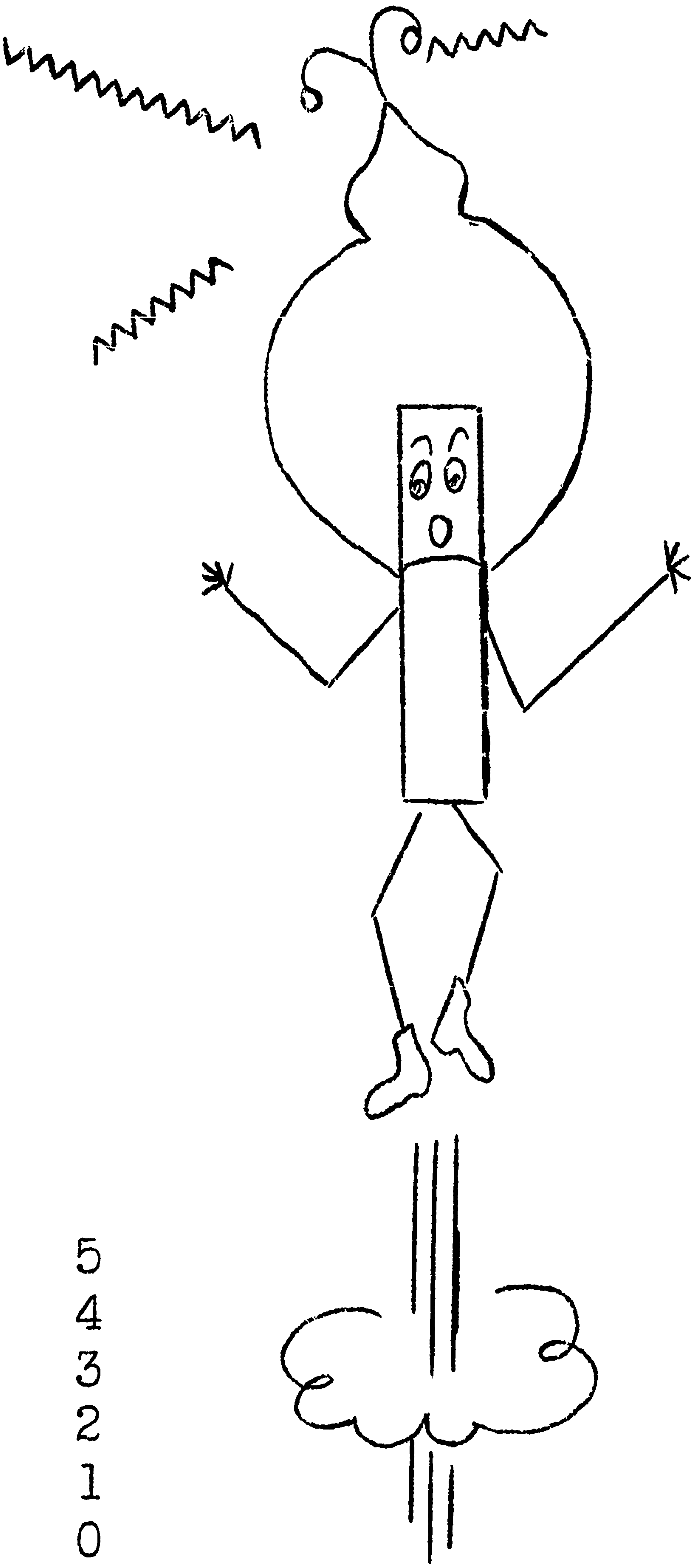

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preface

PREFACE

The purpose of this guide is to help give direction in the use of Cuisenaire materials for developing mathematical concepts, and to aid in the clarification of existing Cuisenaire publications. This work is based upon actual classroom experience by several teachers in the University City Public Schools, talks with other teachers and administrators who have used these materials, attendance at Cuisenaire demonstration classes, and reading and studying books and pamphlets written about this material.* The authors of this guide have made an effort to be as succinct and at the same time as lucid as possible: We do not claim that the Cuisenaire materials are a panacea for all the problems of arithmetic instruction. However, we strongly believe that these materials can be used vividly to develop many mathematical concepts.

An attempt has been made to write these suggestions in a style such that the Cuisenaire materials, though developmental, may be introduced for the first time at any grade level. The swiftness with which a teacher moves through the early exercises will depend on the mathematical maturity of the students.

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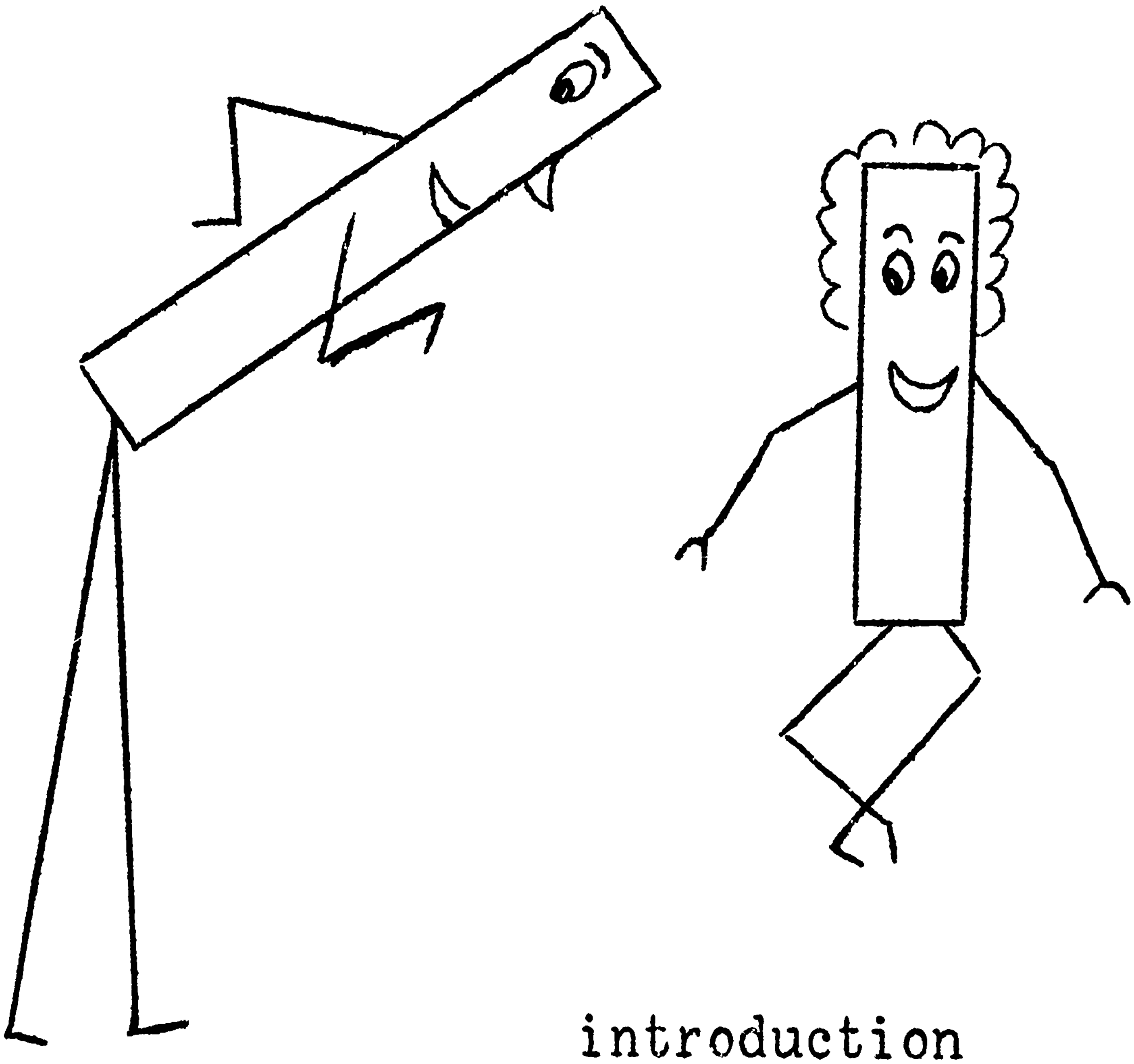
* See Bibliography



introduction

INTRODUCTION

The teacher's attitude toward mathematics is basic to the understanding and use of Cuisenaire rods in the elementary school classroom. "Too often we have tended to underestimate the ability of small children to discover, understand and use basic mathematical concepts from the beginning of their school experience.*

The use of the Cuisenaire rods is considered as one important development of better elementary mathematics teaching. Other manipulative and explanatory devices may also be used in the total program. For example, liquid measure can best be taught by use of pint or quart measures; time, by use of clocks; and money values, by use of real coins.

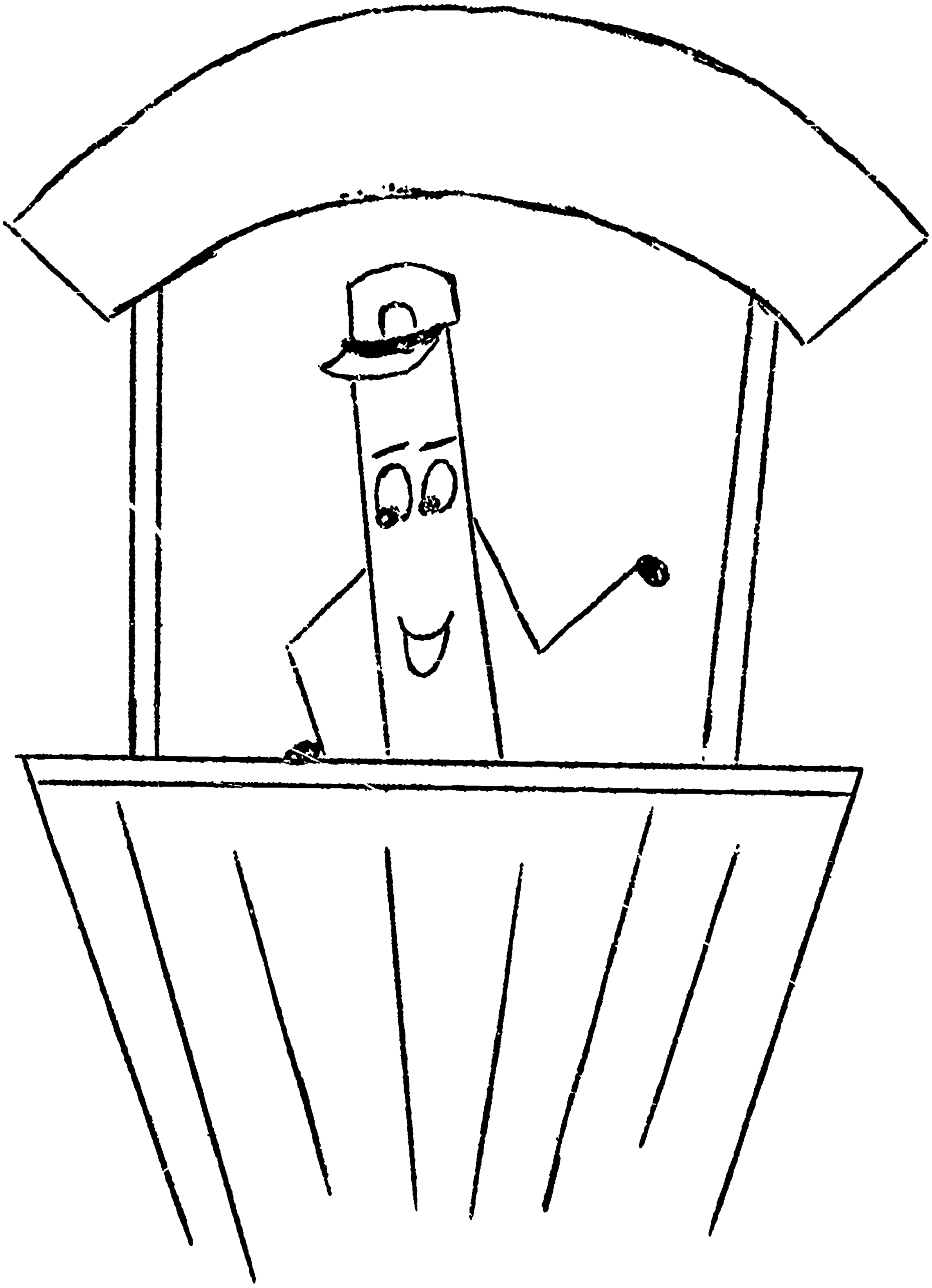
Since the Cuisenaire materials are developed and taught in logical ungraded sequences, this system is readily adapted to our University City ungraded primary. For example, some teachers using the Cuisenaire rods have taught multiplication, division, and beginning fractions in Primary One; while other teachers have introduced basic addition and subtraction in Primary Three - depending on group or individual needs. Cuisenaire rods make it possible for mathematical processes to be introduced at an earlier age. There is a diminishing need for the use of rods in the intermediate grades if they have been used K-3. However, teachers of 4 through 6 will find the rods useful with all children in reinforcing understanding and in developing new concepts.

How the teacher uses the materials within her classroom is regulated by the amount of Cuisenaire materials available and by the needs of the group. They can be used equally well by the whole class, a small group, or by an individual child.

*Rasmussen, Lore; Notes to Teacher, Learning Materials Inc., 100 East Ohio Street, Chicago, Illinois.

The mathematical learning gained by each child as a unique individual is an important aspect of the Cuisenaire program. Within any group there will be children discovering several different ways of answering the same problem by using the rods. Children of all intellectual levels, all emotional and personality makeups can more readily work toward their mathematical potential through experiences with the rods.

Emphasis on computational skills, repetitious drill, and monotonous rote learning are minimized in the Cuisenaire program. Rather, the children have the delightful experience of thinking about relationships which they can see, feel, or manipulate with the rods. The end result will be individuals who enjoy and understand mathematics at their own developmental level.



general information

GENERAL INFORMATION

It is more economical to buy the Basic Classroom Kit designed to equip a class of 25 children. This kit contains 25 bags of Cuisenaire rods (69 pieces in each), a set of books for the teacher's use, and a pack of product cards. It is priced at $47.50. Additional bags and rods may be purchased if needed.

TEACHER PREPARATION

In addition to reading the recommended books in the bibliography, the teacher will feel more secure if she has done some outside experimentation with the rods herself. Pages 14-18 in the Teacher's Commentary show examples of presentation of several lessons, and a developmental sequence is found in Mathematics in Color books. Group work is exciting as children learn to verbalize their discoveries. Written practice may follow an oral lesson. Discovering and creating is fun. Jump in! Enjoy yourself!

CARE

Treat the bag of rods the same as you would a new textbook. Discuss the care of the rods: 1) How easily the small rods could be lost; 2) The importance of putting all the rods back in the bag; 3) Checking the floor under desks after use.

Rods are colored with non-toxic vegetable dye, so are harmless, yet should not be put in mouths for safety and sanitation reasons.

Numbering bags and letting the child be responsible for his bag is one method used to help control carelessness.

COLORS AND COLOR NOTATION SYMBOLS

w - white
r - red
g - light green
p - purple
y - yellow
d - dark green
k - black
n - brown
u - blue
o - orange

Since these symbols will be used daily until replaced by numbers, the children will soon have them firmly fixed in their minds.

Each child may keep a folder for Cuisenaire work. These symbols can be copied with color words and the page used for reference. A large chart could be placed in the room labeling color by letter.

SIZE OF RODS

The rods vary in length from 1 cm. to 10 cm. with a cross section of 1 sq. cm. Games of recognition by touch can be played to help children become familiar with relative sizes of rods.*

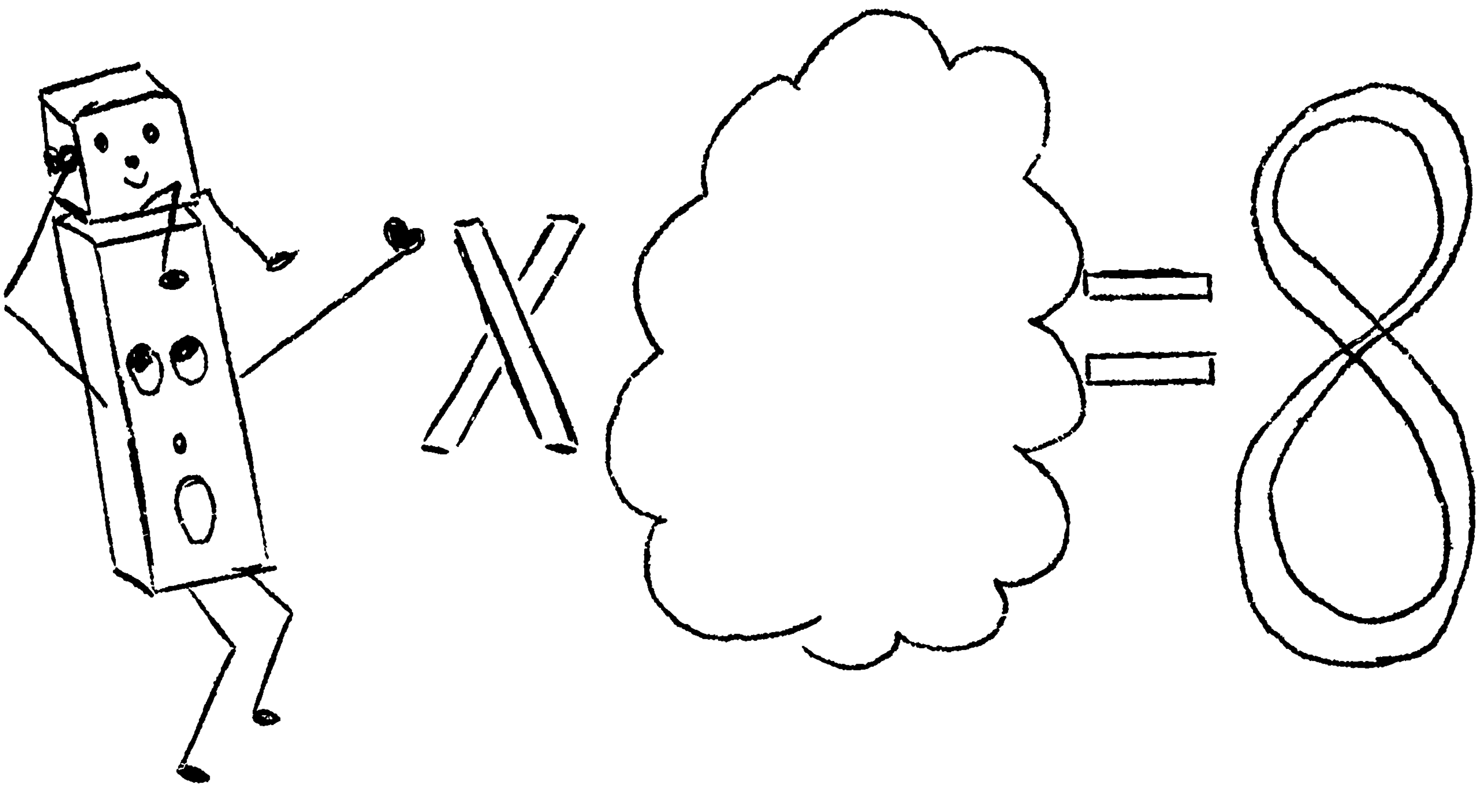
FAMILIES

There are 3 families, the red, yellow and blue-green, which have an interesting relationship in respect to their lengths.** The red family consists of red, purple, and brown. (Colors deepen as length increases.) The yellow family consists of yellow and orange. The blue-green family consists of light green, dark green, and blue.

Two rods - white and black - do not belong to these families although the white rod technically belongs to each.

* See Extended Activities

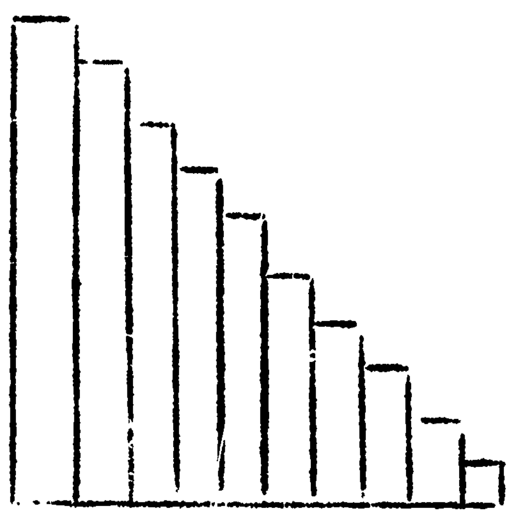
** B-8 p.32



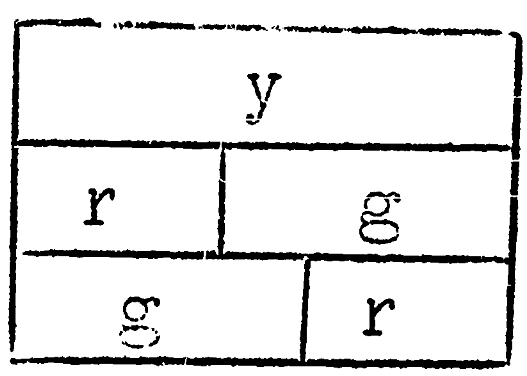
developing mathematical concepts

A. INTRODUCING CUISENAIRE MATERIALS TO THE STUDENT

Teachers who have used Cuisenaire rods agree that when a child is first introduced to the rods he should be allowed a time for free play.* Free play gives the child an opportunity to design, invent, construct; that is to create something. Free play enables the teacher to observe the degree of creativity and insight each child may evidence. The amount of time permitted for free play will certainly vary according to the classroom situation. The teacher should survey carefully the designs which are being created by the children and be aware of those which lend themselves to a mathematical analysis. For example, some child will usually make the staircase design which, when rods have been given number names, will help to order the first ten counting numbers. Another child might make a simple design which leads to the profound mathematical principle of commutativity for addition. Numerous other mathematical ideas derived from free play are possible.**



Staircase



Commutativity
(See vocabulary)

*B-1, pp. 25-29
B-2, pp. 5-6
B-3, pp. 1-4

**B-1, pp. 69-75

B. THE NUMBER CONCEPT AND COUNTING

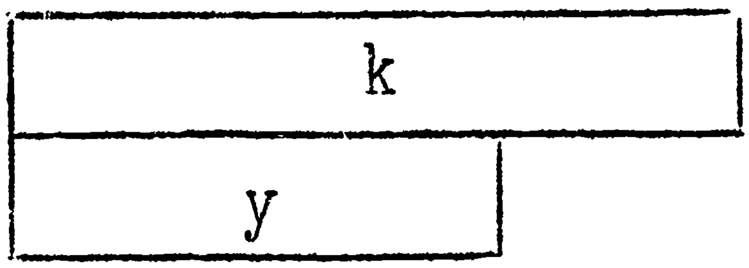
Some people believe that man is born with a number sense and this may well be true. However, teachers know that some children enter school possessing very limited number concepts. Even though a child can count to ten, he may not understand the concept of "threeness". Primary teachers have always done an excellent job teaching concepts such as "threeness" by using blocks, stones, fingers, etc. Cuisenaire rods do not offer anything too new at this developmental stage but may be used as follows: place several sets of rods before the children so that each set contains the same number of rods. Ask the child what it is that these sets of rods have in common. The answers may be very surprising! In this manner they can form the number concepts of three, five, nine, etc., which is a major step in the direction of counting but does not in itself consitute counting. The child at this point may be able to tell "how many" things there are in a collection without really thinking about the numbers as being ordered.

Cuisenaire rods offer an excellent method for ordering the numbers; a method which the child will probably discover for himself. If he names the small block "one", the staircase design and the insight of the child will, with a little direction, do the rest. He now has the concept of counting; i.e., starting with one and going next to the number which is one "bigger", etc. He also has ten small rods very neatly "bundled up" in the orange red, which is very useful in developing the concept of place value and really gets the child to <u>see</u> the meaning of a number such as 17.

O=1 ten	k=7 ones

Counting by twos, threes, fives or tens is easily handled with Cuisenaire rods. In these days the child may even want to count backwards!

As previously mentioned, implicit in counting is the concept of ordering; i.e., an awareness that a given counting number is less than its successor. Very early it is possible to establish the concepts of less than ($<$) and greater than ($>$). Since there are probably more inequalities than equalities in the real world, these are very important concepts. And they are so easy to visualize with Cuisenaire rods!



$7 > 5$
$5 < 7$

C. CONCEPTS OF ADDITION, SUBTRACTION, DIVISION, MULTIPLICATION OF WHOLE NUMBERS.

During free play with the rods the children have experienced relationships which adults know as numbers, and without being aware of the fact, have "discovered" for themselves some of the basic laws of mathematics.

At first the play is without verbalization; then color words are used, and "blue" becomes the name of a particular rod. Later the word "nine" is met as another name for this same rod. It is important to note that although each rod is here given a particular name, children will soon discover the rods can take on other number names.*

After children have experienced "pattern-making" for different colored rods, the + and = signs are introduced, and patterns may be recorded.

Example: w+r = g   w+w+w =g
r+w = g

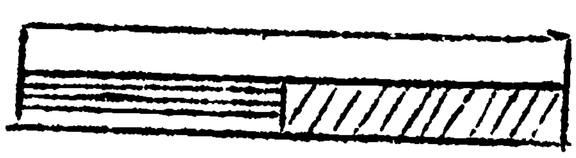
By the time that the numerals are introduced the child will be familiar with such statements as:

An orange rod is equal to two yellows
If the green rod is one, the blue is three
Five red rods make one orange rod.

He will also have counted the rods when in the pattern of the staircase. White will have been called "one", red "two"...., etc.,

Addition is taught by placing rods end to end (called a train), then using a longer rod for measurement.

Example: Green placed end to end with green measures the length of dark green, i.e., g+g = d, read as green plus green equals dark green.

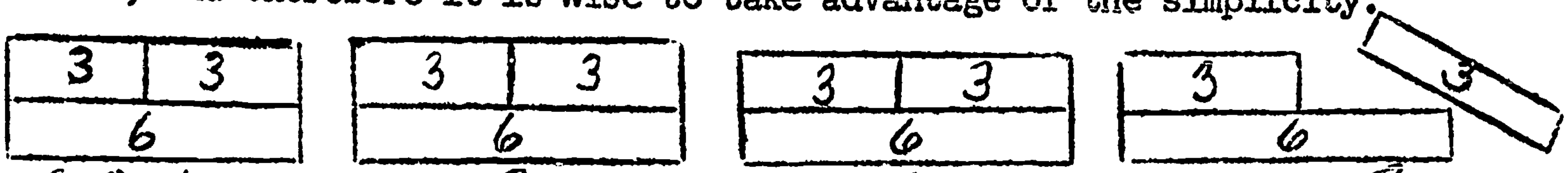


(All rods laid flat on table)

* B-3, page 7

Concepts of whole numbers, cont'd.

It is very easy to teach several of the processes of mathematics at one time, and therefore it is wise to take advantage of the simplicity.



$3+3=6$ $2\times3=6$ $6\div3=2$ $6-3=3$

In subtraction the term minus is used. The smaller rod can be placed on top of the larger rod and the remainder found by finding a rod equal to the difference between the length of the two rods.

The dot (•) or times (X) are used at first as "of the" (e.g. 2 of the 2's). When the child discovers 3+3 = 6, he can also see 2 X 3 = 6 and 6 ÷ 3 =2 as well as the conventional 6 - 3 = 3.

Numbers made of multiples of 2 (or having a factor of 2) are known as <u>even</u> numbers. Other whole numbers are <u>odd</u>. When we add two even numbers the result is <u>even</u>. Older children enjoy discovering this principle and proving it.

When you add 2 even numbers = even as 2 + 6 = <u>8</u>
When you add 2 odd numbers = even as 1 + 3 = <u>4</u>
When you add even + odd " = odd as 2 + 1 = <u>3</u>

	E	O
E	E	O
O	O	E

E = even numbers
O = odd numbers

When you multiply 2 even numbers = even as 2 X 4 = <u>8</u>
When you multiply 2 odd numbers = odd as 3 X 5 = <u>15</u>
When you multiply odd X even numbers = even as 3 X 4 = <u>12</u>

X	E	O
E	E	E
O	E	O

Concepts of whole numbers, cont'd.

The orange-red allows place value to be taught easily and meaningfully. It also makes the teaching of borrowing and carrying a visible process.

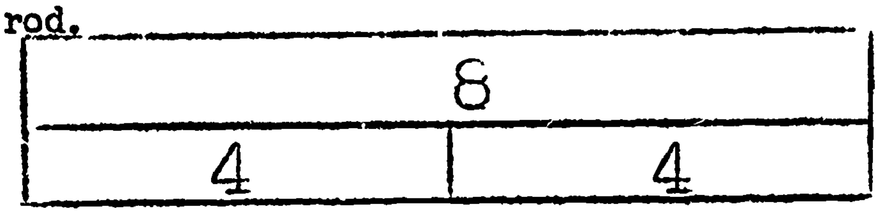
Story problems and verbal problems can be used from the very beginning.

The operations with whole numbers are so closely related to operations with fractions that the beginning concepts of fractions are introduced early.

D. FRACTIONS

The first major extension of the number system is the introduction of fractions. It is easy to introduce the fraction concept in a concrete, meaningful, and mathematical manner by using Cuisenaire rods. The idea of a fraction as a relationship found by comparing two counting numbers in a particular manner can be clearly illustrated. If for example, you consider the brown rod (8) and the purple rod (4) you see immediately that the purple rod is one-half[1] of the brown rod.

1/2 of 8 = 4

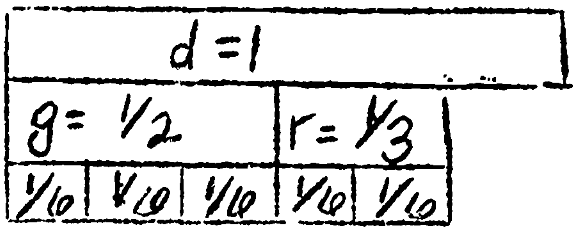


You can now find a large set of number pairs which have this same one-half relationship; e.g., 1/2, 2/4, 3/6, 4/8, ... If you now replace the "of" by a times sign, which is a logical thing to do in view of preceding comments, you find the product of a fraction and a whole number. This idea can easily be extended to work with 1/3, 2/3, 3/4, etc.*

Consider for a moment a class of 30 children. Suppose this class has studied fractions for quite some time. What would be the result if the teacher asked each child to write a number (or numeral if you like) on a piece of paper? All whole numbers? Twenty-five whole numbers? Could it be that the youngster is a bit dubious about a fraction being a number? An excellent way to get the child to beieve that a fraction is a number is by renaming the rods.* If the dark green rod is "one", then the child will usually, with great haste, want to name the red rod "one-third". The child may now want to try his luck at adding, say 1/2 and 1/3.

1 The teacher may be required to introduce the name "one-half".
* B-3 p. 7

D. Fractions cont'd.



It is also easy for the child to see that $1/2 > 1/3$.

If d =1, then g = 1/2. r = 1/3. 1/3 < 1/2.

As you see, the concepts of equivalent fractions and order among the fractions can be developed early. Also, solving problems such as 1/2 of 6, 2/3 of 9 and 2/5 of 10 are successfully handled by rather young children. As you move along in the development of concepts of fraction by a fraction.** The following discussion is a possible procedure for teaching multiplication of fractions.

Suppose the black rod is named "one", then the white rod is 1/7, the red rod 2/7, etc. Consider the problem of finding 1/3 of 3/7 (or 1/3 x 3/7). Under this present naming scheme, the light green rod is 3/7 and it is easy to see that 1/3 of the light green rod is the white rod. But the white rod is 1/7. Therefore 1/3 x 3/7 = 1/7. You now get some practice doing exercises such as 1/5 x 5/7, 2/3 x 3/7, 4/5 x 5/7, etc. To be a little creative, ask the child to make

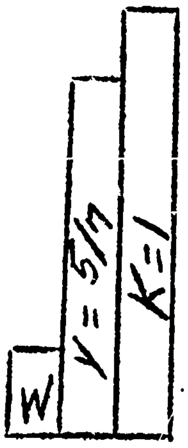
1/3 of 3/7 = 1/7

$\triangle \times \square = 1/7$ a true statement by filling in the appropriate fractions. Analyze this problem by using Cuisenaire rods. You want as an answer the white rod (1/7) which looks like this picture.

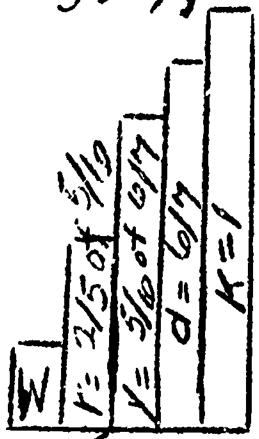
** B-8 pp. 14-15

D. Fractions, cont'd.

If you now place the yellow rod between the white and black rods, you see that one possible solution is 1/5 x $\boxed{5/7}$ =1/7. Watch the creative child go to work on this one: He will probably discover that 1/2 x 2/5 x 5/6 x 6/7 - 1/7 (see second diagram on this page). He made this discovery by inserting several rods between the white and black rods.



1/5 of 5/7 = 1/7



1/2 of 2/5 of 5/6 of 6/7 = 1/7

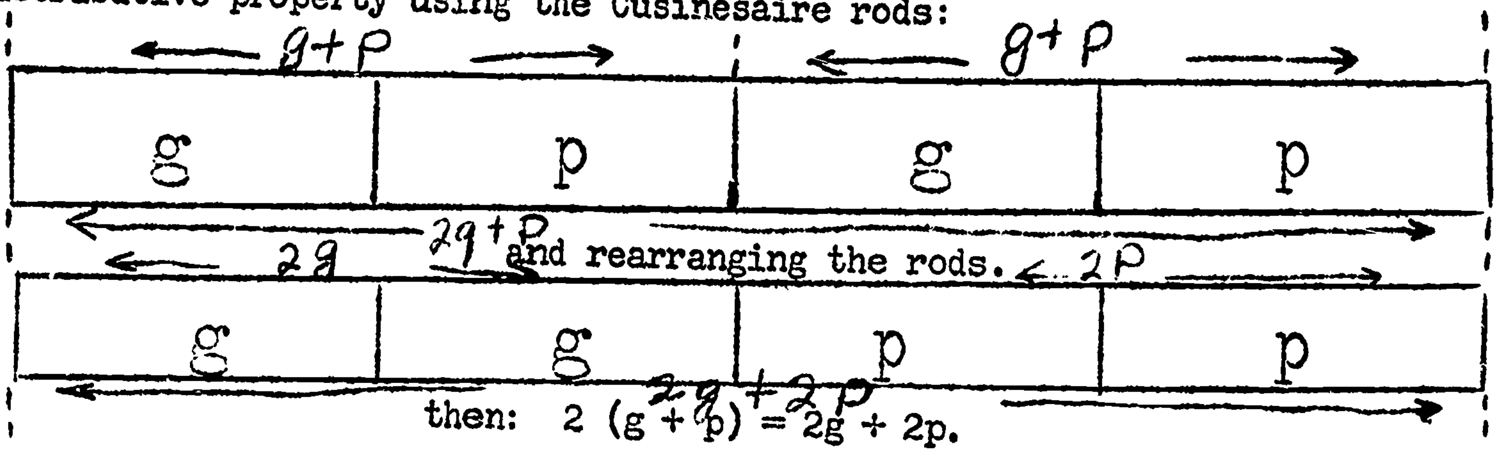
You have probably noticed that in the preceding exercises the denominator of the first fraction is the same number as the numerator of the second fraction. How would you handle 2/3 x 5/7? The concept of equivalent fractions has been previously established; e.g., 2/3 = 4/6 =8/12 = 10/15 and 5/7 = 10/14 =15/21 = 20/28. Therefore 2/3 x 5/7 = 10/15 x 15/21. But 10/15 of 15/21 = 10/21. So we have 2/3 x 5/7 = 10/21. Some children may now be ready to state a rule.

Since concepts of fractions require a span of time to develop they should be introduced early and handled carefully. Cuisenaire rods are very helpful in developing several of the basic concepts involving fractions.

E. OTHER IMPORTANT CONCEPTS

1. Distributive property

The distributive property, an extremely important mathematical concept, is easily visualized by using Cuisenaire rods. You know that $2 (3 + 4) = (2 \times 3) + (2 \times 4)$ since the expression to the left and the expression to the right each name the number 14. Here is an example of the distributive property using the Cusinesaire rods:



2. Square numbers

Focusing attention on the white rods, you see that certain numbers of these rods can be placed in a square array, e.g., 4,9, and 16. These are called square numbers. Square numbers seem to fascinate children and they will eagerly look for more. The value here is that many of the children remember the square numbers and immediately know the multiplication facts 2×2, 3×3, ... 8×8.

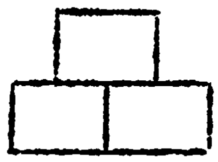
3. Rectangular numbers

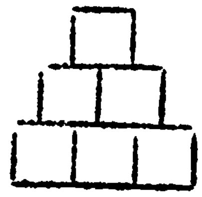
Six white rods cannot be placed in a square array, but can be placed in a rectangular array having length and width each greater than one. (It is necessary for the rectangle formed to have length and width each greater than one.) The number six is an example of a rectangular number. Some numbers of white rods; e.g., 7, cannot be placed in this type of rectangular array. <u>Composite</u> and <u>prime</u> numbers can be introduced in this manner.

4. Triangular Numbers

If there are square and rectangular numbers, then why not triangular numbers? Find a number of white rods which will form a triangular array. Three and six are a pair of numbers which will accomplish this feat.

3 rods





6 rods

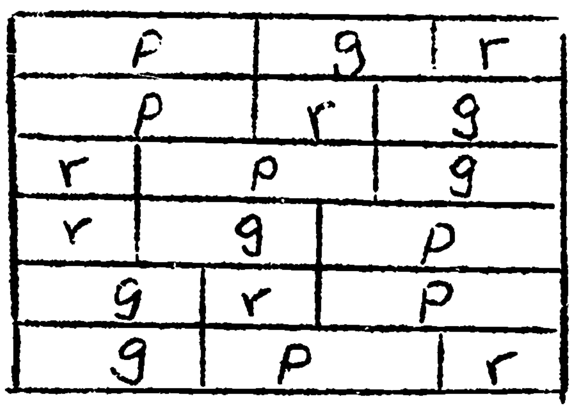
It is interesting to note that the sum of these two triangular numbers is a square number. Could this be true for any two consecutive triangular numbers?

5. Area and Volume

Cuisenaire rods have been used successfully to aid in teaching the concepts of area and volume.*

6. Permutations

Many times children will make a pattern by placing three different colored rods end to end in as many different ways as possible.



What they have done is find the number of permutations of the three rods. This is an important concept in higher mathematics.

The list of concepts which can be developed using Cuisenaire rods may be endless. Who knows?

*B-4, Book C, pp. 17-58
B-1, pp. 48-51

F. EXTENDED ACTIVITIES

From early play experience to problem solving, Cuisenaire rods can be used for many types of extended activities. The variety and scope of these activities depends upon the creative imagination of the teacher and/or the pupils. Only a few typical examples can be given here. Many other unique activity experiences will be discovered as the teacher and class work together.

GAMES

TOUCH-AND-TELL - Each child puts his hands behind him and is given two rods of unequal size. By sense of touch only, the child finds and holds up the larger or smaller one as directed. Later three rods can be used. Eventually the child can have some of each color in a bag and pick out by touch the white, the orange, the red, or any rod, or group of rods as requested. With two or more of the same lengths in the bag, the child can pick out sets of "twins" (2 red, 2 yellow, etc.)

MAGICIAN - A child holds up a single rod, as yellow, and the magician says, "I can change it to green and red!" as he holds up the two rods. (This can be done with many variations.)

PRACTICAL CLASSROOM SITUATIONS (PRIMARY)

ADDRESS AND PHONE - Children learning these facts can show them with rods and learn to read each others' addresses and phone numbers.

MAKING TEAMS - Children can quickly figure out size of teams in any classroom games.

LUNCHROOM MONEY PROBLEMS - How much for a plate lunch, milk, or dessert with change from a dollar can be quickly solved with the rods.

VISUALIZING TEXTBOOK LESSONS (INTERMEDIATE)

In the intermediate grades such subjects as perimeter, volume, graphs, number lines, decimals, profit and loss, fractions, and many other topics can be graphically illustrated with the rods.

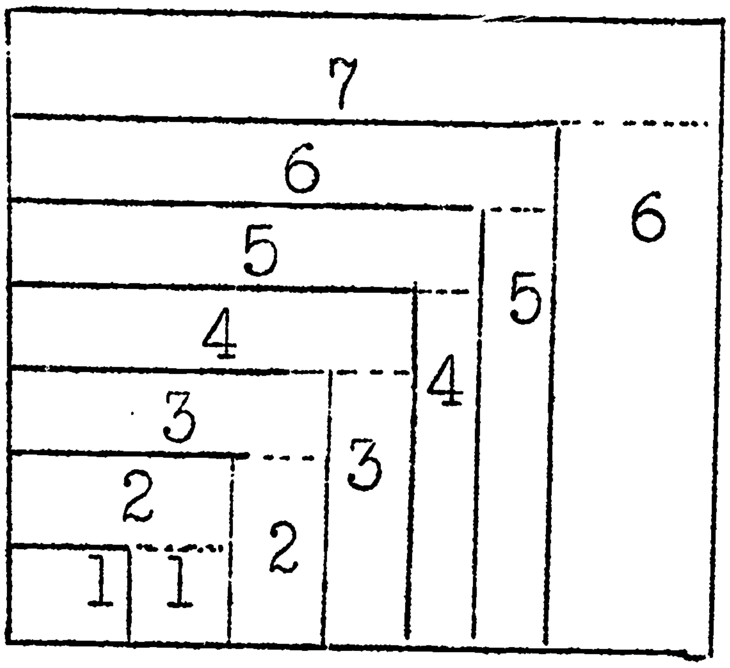
Extended Activities, cont'd.

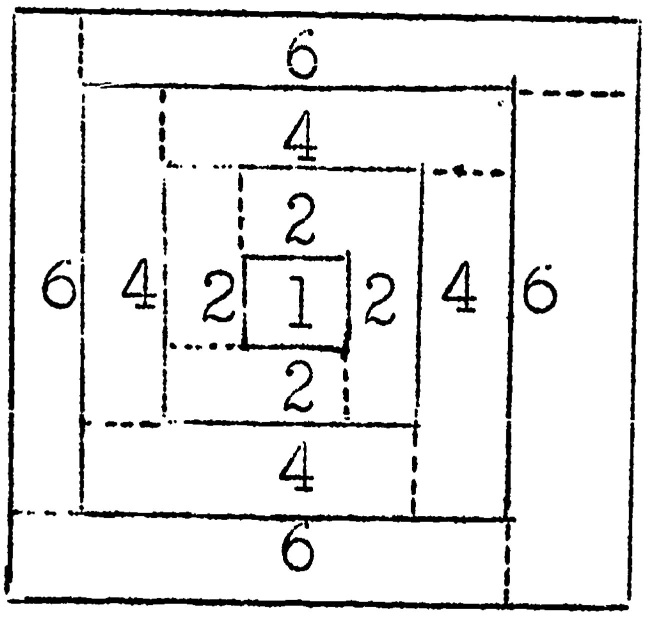
DRAWING ACTIVITIES

Using squared paper the children can draw and label with color or number names the various color rods in individual pictures or in successive staircase designs. Permutations or creative designs may be discovered by the children.

The teacher can give the children mimeographed drawings of rods (accurate in proportion) and let the children label or color them.*

At more advanced levels, interesting number squares and mathematical designs can be created. What is the mathematical significance of the following patterns?





*B-2, page 41-42

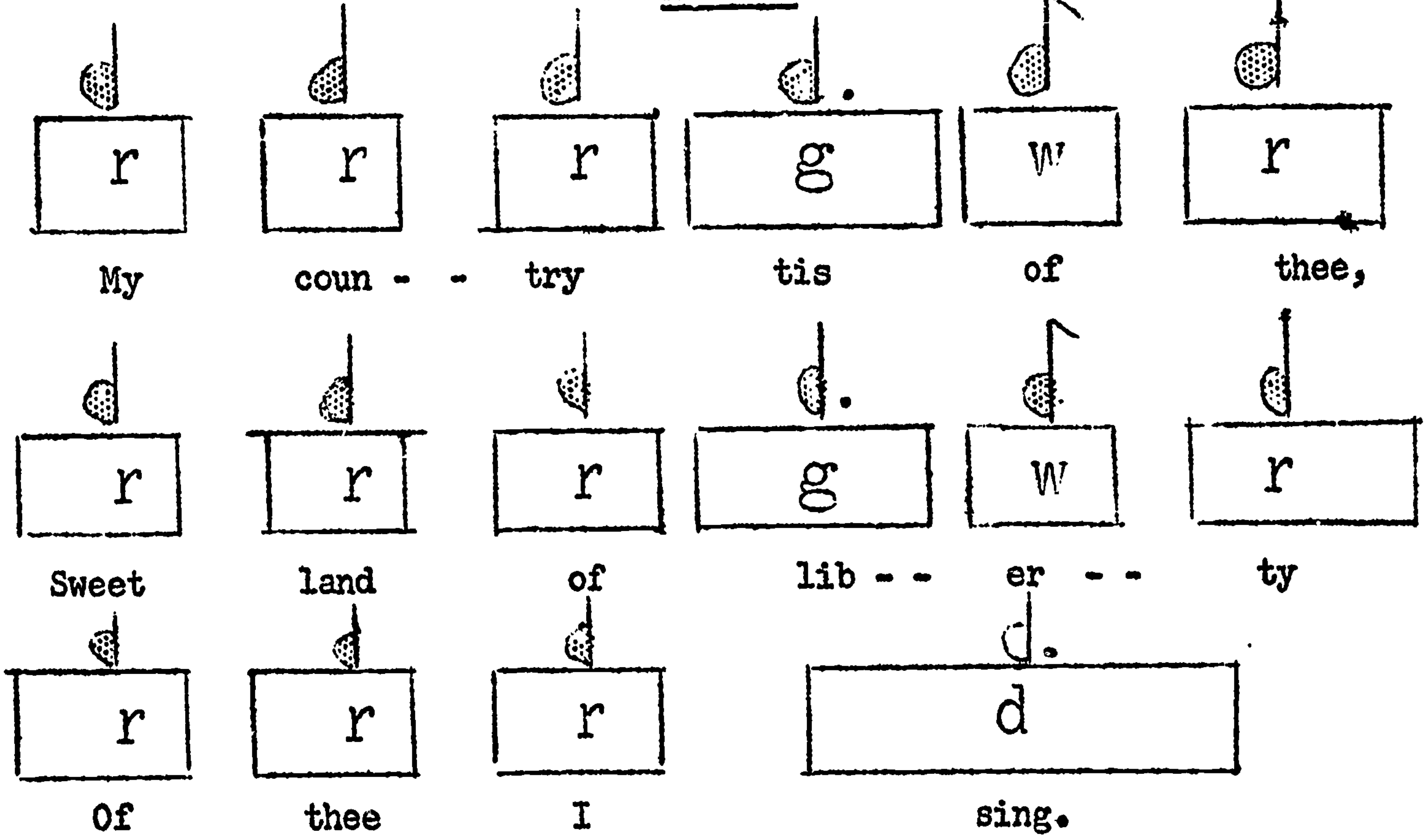
Extended Activities, cont'd.

RHYTHM ACTIVITIES*

Songs with regular rhythm patterns can be worked out with rods, giving each note a rod value, as

eighth note	♪	= white rod
quarter note	♩	= red rod
dotted quarter note	♩.	= green rod
half note	𝅗𝅥	= purple rod
dotted half note	𝅗𝅥.	= dark green rod
whole note	𝅝	= brown rod

AMERICA



Children can work these patterns out on their desks, or work them together to make a rhythm chart of the song for the room.

* B-6

SAMPLE

Try to form the length of any one rod by using only red rods; light green; purple, yellow. Can you always do it?

	red	l. green	purple	yell
white				
red				
light-green				
purple				
yellow				
dark-green				
black				
brown				
blue				
orange				

Which rods can be covered by using only red rods? Light green? Purple? Yellow? Which rod do you need to make up the length of the rod you started with when you can't do it with the red? the light green? the purple? the yellow?

Make trains using only dark-green rods and only black rods. Can they be the same length?

Make trains that are all brown and all orange. Can they be the same length?

1 **Transition to numerals** SAMPLE

r+w = □ w + r = □
2+1 = □ ___+___ = □

g+r = □ y + p = □
3+2 = □ ___+___ = □

2

p = □ 2+2 = □
g = □ 2+5 = □
d = □ 3+3 = □
y = □ 1+4 = □
w = □ 2+4 = □
r = □

Addition - Subtraction - Fractional parts

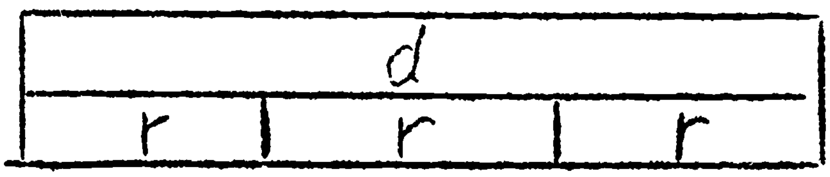
4+3+1 = □ 5+5 = □ 4 = 1/2 • □
11 - 7 = □ 5 = 1/2 • □ 2 = 1/2 • □
10 - 5 = □ 8-4 = □ 2 = 1/4 • □
4+4 = □

Multiplication - Division

4=2x2, So 2 = $\boxed{1/2}$ x 4 and 4÷2=2

How many red rods make 6, $\boxed{3}$, so 6÷2=3

3x2=6



1/3 x d

3=1/2x6

2=1/3x6

More complicated equations

(2/3x12)+5/7x(14-7)+2 = □

4/7x(15-8)+3/5 x(15-10) = 15- □

Match words to the symbol

equal fraction
× =
brackets of the
½ ¼ [ ]

True - False?

3/8x16 = 6/16x16?
2/8x16 = 1/4x16?
1/2x14 = 5+3?
8/12x12 = 3/4x12?

SAMPLE

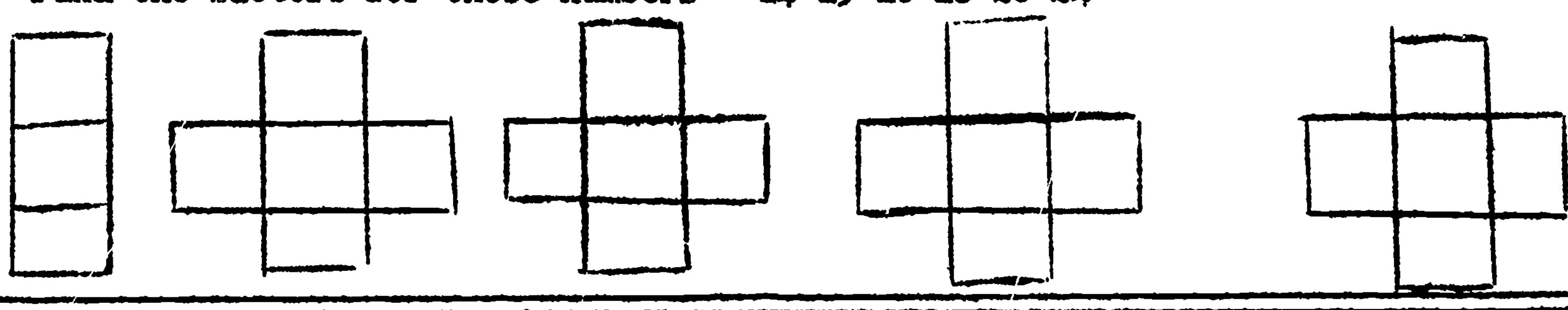
Use of open sentences
Use of parenthesis

$1+1+\square=4$	$(3\times1)+2=\square$
$3+\square=5$	$4-(2+1)=\square$
$3+\square+1=5$	$3+(1/2\times4)=\square$
$5-(2+1)=\square$	$(1\times1)+(1+4)=\square$

Introduction to fractional parts

r= $\square$	y= $\square$
2= $\square$ w's	5= $\square$ w's
w=1/2 • $\square$	w=1/5x $\square$
What part of the	2=2/5x $\square$
green rod is red? $\square$	3/5x $\square$ =3
What part is white? $\square$	

Find the factors for these numbers - 14 15 16 18 20 24 - Color them below:



Compare and circle the larger one:

1/3x15 and 2/5x15
2/5x15 and 4/5x15
1/3x15 and 6/15x15

Story Problems

S A M P L E

1. If you have ten marbles, how many can you give to each of:

   10 boys____________________
   five boys____________________
   two boys____________________

2. How many marbles would you need to give one each to nine boys?__________
   Two each to 4 boys?_______________
   Three each to 3 boys?_______________
   Five each to 2 boys?_______________

3. Two boys and one girl pick flowers in a garden. The boys have three each and the girl has four. How many flowers do they have altogether?__________

4. When I gave away half of the marbles I had, I still had five left. How many did I have in the beginning?_______________

5. Sue treated her friends to grape juice. Her mother asked her to put an ice cube in each of the five glasses. The ice tray held 12 cubes. How many were left in the tray?_______________

6. A Boy Scout troop gathered old newspapers to raise money for a camp fund. One group gathered 40 pounds and the other group gathered 34 pounds. How many pounds did the two groups gather?_______________

7. Each pound is worth 2 cents. How much money did they receive for it?______

   Think of the words triangle, tricycle, triceritops and tripod. What does the tri in these words mean?

   Think of the words bicycle, binoculars, biweekly and biplane. What does the bi in each of these words mean?

* * * * * * * * * * * * * * * *

1. We have 26 children in our room this year. One day only 19 were present. How many were absent?__________
   If 14 of the children in the room are girls, how many are boys?__________

2. June has 30 days. How many whole weeks?______How many days are left over?________ If you were to spend one-third of June on a vacation trip, how many days would you be gone?______On what day of the month will June be half-gone?_______________

3. If pencils are 2 for 5¢, how many can I buy for 20¢?____________

4. If erasers are 2 for 3¢, how many can I buy for 15¢?____________

5. For 40¢ how many pencils and erasers can you buy?__________Find as many answers as you can, with no more than 2¢ change left.

SAMPLE

Use of > and <

Make these sentences TRUE.

< means less than (6<7)

> means greater than (7>6)

= equal to

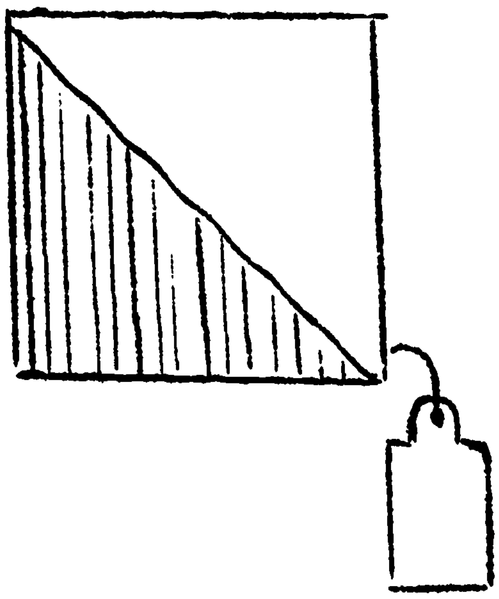
10	=	9+1	3+27	>	27+2
17		1+7	10+9		7+10
3+5		9	79-1		77+1
0		2-1	3+3+4		3+5+2
7-7		5-5	5+7+1		7+5+1
4+4		6+6	10-2		6+2
2+3		3+2	2x3		3x2
20		10+10	1/2x6		3+0
38		30+10	14+7		5x4
45		22+30	9+6		10+5
20+4		25-1	11-7		1/2x8

S A M P L E

Fractional Parts

Using your rods, find the value of these figures:

A



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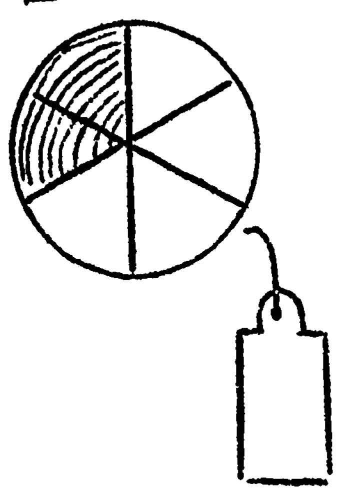

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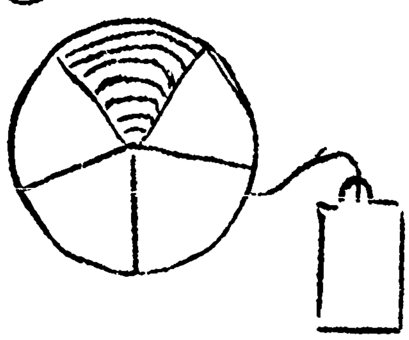
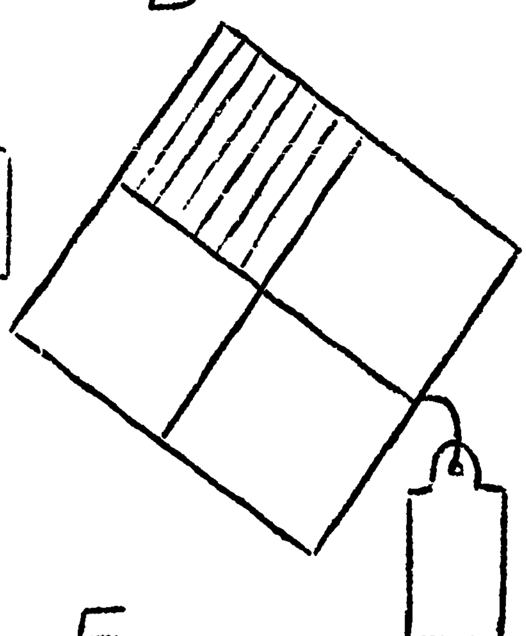
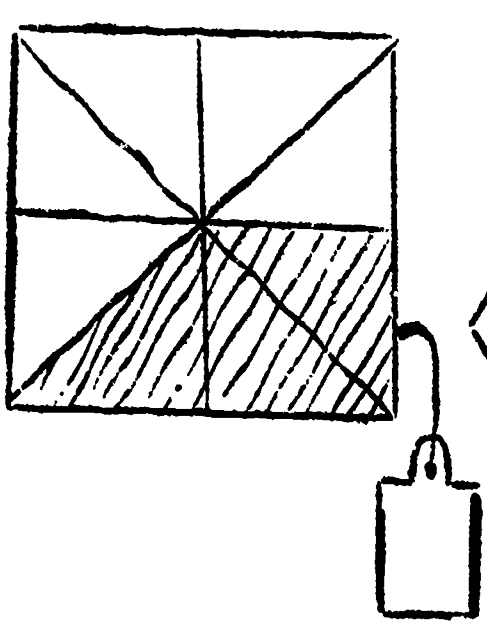


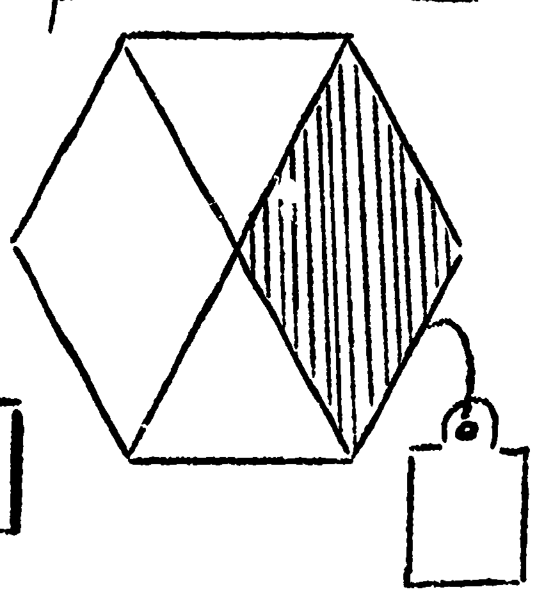
D



E



F



	Dark	White	Whole Tag
A	3		
B	3		
C	3		
D	3		
E	3		
F	4		
B			6
A			16
C			20
D			16
B		20	
C		20	
E	30		
F		24	
A			100
C	6		
B		16	
D	30		
E		30	
F		48	

SAMPLE

Excerpts of Problems

Adding Like Fractions

$1/4 + \square = 2/4$ or $1/2$

$1/5 + \square + 2/5 = 4/5$

$3/8 + 2/8 = \square$

$1/2 + 5/12 + \triangle = 7/12$

$2/5 + \bigtriangledown = 3/5$

$\square + 2/7 + 3/7 = 6/7$

Adding Mixed Numbers

$1\,1/3 + 1\,1/3 = \triangle$

$2\,2/7 + \square = 3\,3/7$

$2\,3/5 + 3\,1/5 = \bigtriangledown$

$\square + 1\,3/8 = 1\,5/8$

$2\,1/4 + \square = 3$

$\square + 3\,1/8 = 4\,3/8$

Subtracting Like Fractions

$2/3 - 1/3 = \square$ $\quad 3/8 - \square = 1/8$ $\quad 6/7 - 3/7 = \square$

Subtracting Mixed Numbers

$4\,3/5 - 2\,1/5 = \square$ $\quad 3\,2/3 - 1/3 = \square$ $\quad 1\,1/2 - \square = 1$

Joe had a board 3 2/3 feet long. He sawed off a piece 1 1/3 feet long. How long was the piece that remained?____________

SAMPLE

These are samples of different types of fractions that are easily understood by children using the Cuisenaire rods. Similar problems may be found in some basic textbooks.

Find the missing numbers

$\frac{1}{3} = \frac{\ }{9}$ $\frac{2}{3} = \frac{\ }{6} = \frac{\ }{9} = \frac{\ }{12}$ $\frac{1}{4} = \frac{\ }{12}$

$\frac{2}{7} = \frac{\ }{14}$ $\frac{2}{5} = \frac{\ }{15} = \frac{\ }{30}$ $\frac{3}{11} = \frac{\ }{33}$ $\frac{1}{2} = \frac{\ }{6}$

Change to eighteenths

$\frac{1}{2} = \frac{\ }{18}$ $\frac{1}{3} = \frac{\ }{18}$ $\frac{1}{6} = \frac{\ }{18}$ $\frac{5}{6} = \frac{\ }{18}$ $\frac{1}{9} = \frac{\ }{18}$ $\frac{5}{9} = \frac{\ }{18}$ $\frac{7}{9} = \frac{\ }{18}$

How much is: $\frac{1}{2} + \frac{1}{3} + \frac{5}{6} + \frac{7}{9} =$ ______

$\frac{1}{2} + \frac{1}{3} = \square$ $\frac{1}{9} + \square = \frac{3}{18}$ $\frac{1}{2} + \square = \frac{11}{18}$

$\frac{1}{3} + \frac{5}{6} = \square$ $\frac{7}{9} + \frac{1}{3} = \square$ $\frac{1}{2} + \frac{1}{9} = \square$

$2\frac{1}{4} + \frac{1}{2} = \square$ $2\frac{3}{8} + 2\frac{5}{8} = \square$ $2\frac{3}{4} + 1\frac{5}{8} = \square$

Find the difference:

$1 - \frac{3}{4} = \square$ $\frac{7}{8} - \frac{3}{4} = \square$ $7\frac{1}{2} - 7\frac{1}{4} = \square$

$2 - \frac{7}{20} = \square$ $\frac{5}{8} - \frac{1}{2} = \square$ $9\frac{7}{8} - 6\frac{3}{4} = \square$

$1 - \frac{1}{6} = \square$ $\frac{7}{9} - \square = \frac{1}{9}$ $3\frac{3}{8} - 1\frac{3}{4} = \square$

SAMPLE

Show the following decimal fractions.
The orange rod is the one rod.
Write the common fraction equivalent.

$.5 = \_\_\_\_ \quad .7 = \_\_\_\_ \quad .2 = \_\_\_\_ \quad .4 = \_\_\_\_$

$.3 = \_\_\_\_ \quad .6 = \_\_\_\_ \quad .1 = \_\_\_\_ \quad .9 = \_\_\_\_$

Add these decimal fractions using the rods

$.2 + \square = .5 \qquad .1 + \square = .9 \qquad .1 + .3 + .4 = \square$

$.4 + .3 = \square \qquad \square + .7 = .8 \qquad .3 + \square + .1 = .9$

Write the mixed number:

$1.3 = \square \qquad 3.7 = \triangle \qquad 2.1 = \square$

Add:

$1.2 + 3.3 = \square \qquad 3.7 + 4.2 + \square = 9.6$

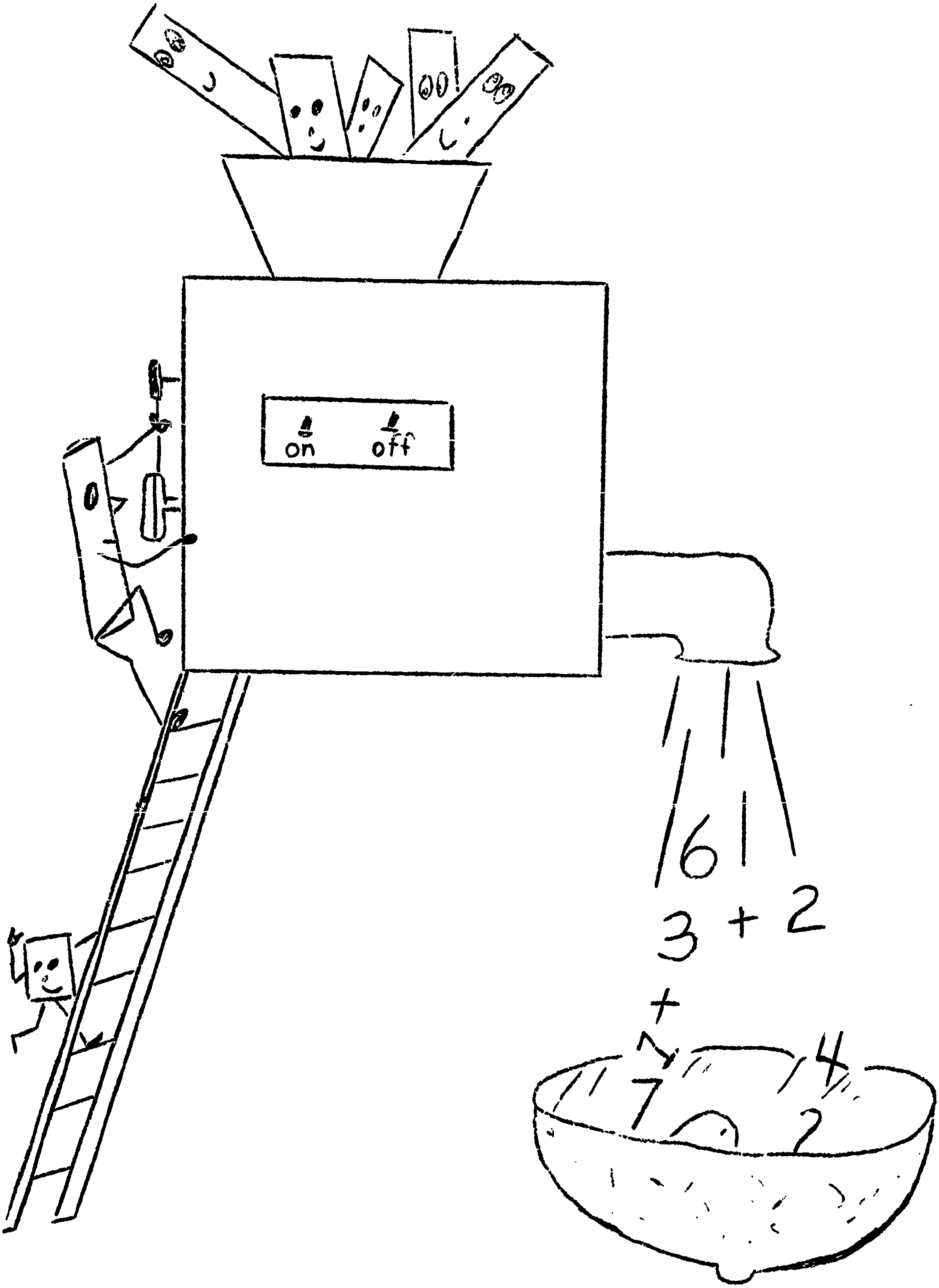
Subtract:

$.7 - .3 = \square \qquad 1.4 - .7 = \square \qquad 3.6 - 1.2 = \square$

Multiply by use of rods:

$\frac{2}{5} \times \frac{5}{7} = \square \qquad \frac{2}{3} \times \frac{3}{4} = \square \qquad \frac{1}{2} \times \frac{2}{9} = \square$

Story problems, verbal problems, and extensions of these samples will probably be apparent.



summary

S U M M A R Y

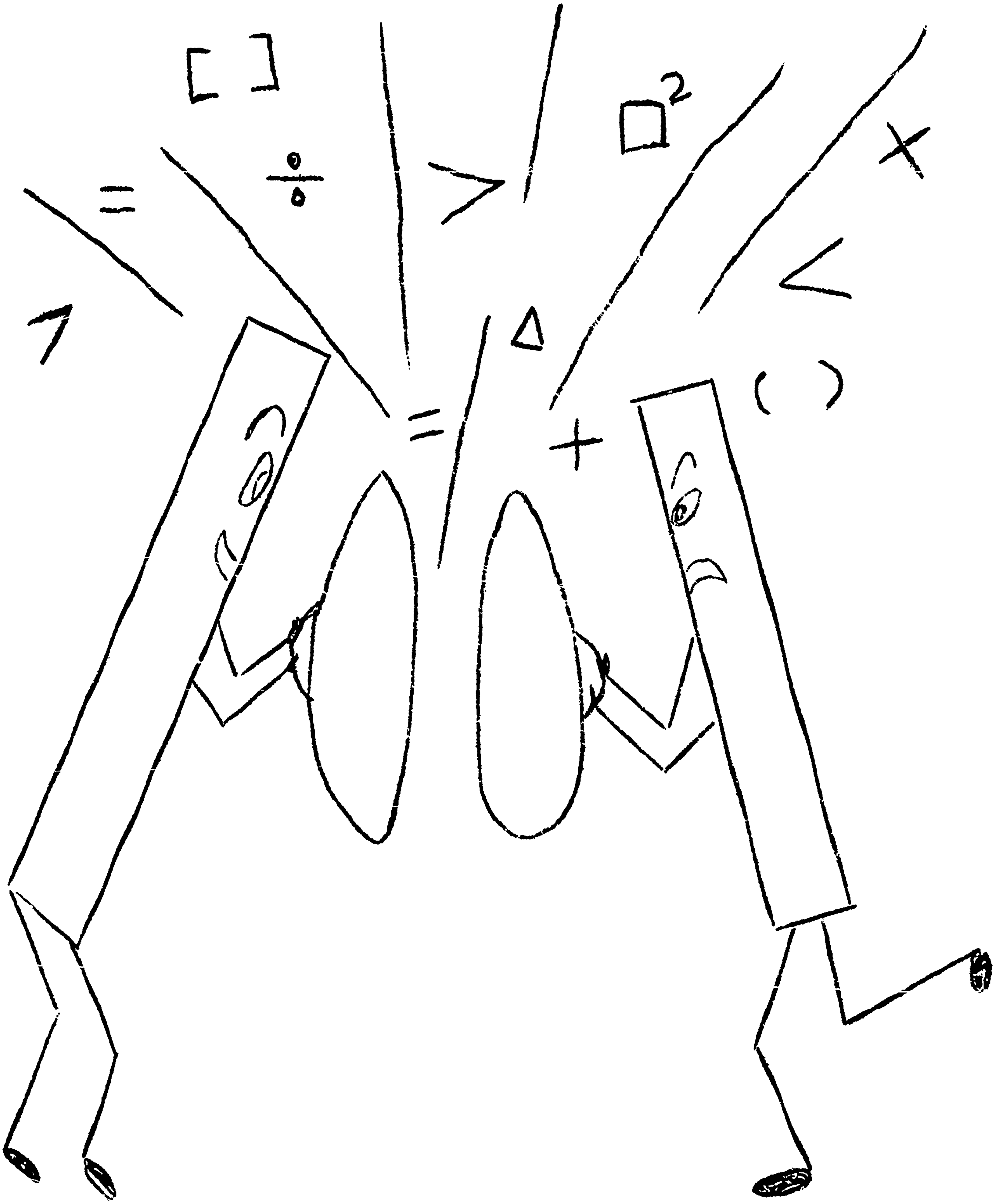
There has been one word used repeatedly by those teachers who have made use of Cuisenaire rods in the classroom = FUN! Children seem to learn more rapidly when learning is fun. Many teachers who have used Cuisenaire materials no longer view mathematics as an esoteric subject, but through discovery as alive, meaningful and necessary.

We* did not go into great detail in this guide and therefore did not emphasize some of the basic principles as much as we would have liked. Principles such as commutativity, associativity, distributivity and estimation cannot be over-emphasized.

It is definitely not our intention to imply that a teacher should try all of our suggestions in one year, but we would like you to give the Cuisenaire rods a try, helping the children discover and prove basic mathematical concepts. Mathematics is a developmental process. Go on your way and you will be amazed at the advancement of your group. Do not advance just to be advancing while leaving behind muddled and confused thought. The important thing is to <u>know</u> what you are talking about and to have a <u>clear</u> <u>picture</u> in your mind.

* Carl Bruns,
Marion W. Green,
Challie Loomis,
Edith McKinnon,
Jean Wolff.

June, 1963.



vocabulary and symbols

SYMBOLS AND VOCABULARY

$+$	Plus - Cuisenaire prefers the use of the mathematical term plus.
$-$	Minus - Preferred instead of "take away".
$\times$ or $\circ$	Times - or "of" as in $\frac{1}{4} \times 12$ or $\frac{1}{4}$ of 12 or $\frac{1}{4} \cdot 12$
$\div$	Divided by - $6 \div 3$ or $\frac{6}{3}$
$>$	Greater than $4 > 2$ 4 is greater than 2
$<$	Less than $3 < 4$ 3 is less than 4
$=$	Equals or is equivalent to
$\neq$	Not equal to
$\square$, $\triangle$, ∇, A, B, $\square$	Various symbols may be used to indicate the place-holders or unknown quantity in an open sentence.
$\square^2$	Any number substituted in the box by itself "squared" or to the "second power". $3 \times 3 = 3^2$ $\square \times \square = \square^2$
$\square^3$	"Box cubed" or any number substituted in the box to the "third power". $4 \times 4 \times 4 = 4^3$ $\square \times \square \times \square = \square^3$

( )	Parenthesis - Children should be taught to compute the portion of an equation within the parenthesis or brackets first. Example: $2 + (\frac{1}{2} \times 2) = \square$
[ ]	Brackets - Parenthesis should be done first, brackets second, and then the remainder of the equation. $\left[\frac{1}{2} \times (2 \times 2)\right] + (2 \times 3) = \square$
... or ⋮	Ad infinitum On and on and on...(could continue indefinitely).

VOCABULARY

Addends - One of two or more numbers being added in an addition problem.

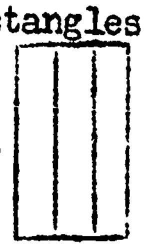
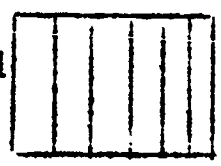
Associativity - Term used to refer to the associative property(see Basic Law).

Cardinal number - A number which tells how many things there are in a group.

Commutativity - A term used to refer to the commutative property(see Basic Law).

Composite numbers - Numbers which have <u>more than 2</u> whole number factors.
Example: 32 has 1, 32, 2, 16, 4 and 8 as its factors.
Therefore, it is composite.

Congruent rectangles - Rectangles that are the same shape and size.
Example:

3 of the 6's  and  6 of the 3's form congruent rectangles.

Distributivity - Term used to refer to the distributive property (see Basic Law).

Equivalent Fractions - Fractions with the same value.
Example: 1/2, 2/4, 3/6, 4/8, 5/10...
2/3, 6/9, 8/12, 10/15, 12/18...

Even number - Whole numbers that may be evenly divided by 2.
Example: 64, 26, and 322.

Factor - The numbers that are being multiplied.
Example: $5 \times 3 = 15$
5 and 3 are the factors of 15

Example: $\square \times \triangle = 12$
3×4 and 2×6 (2, 3, 4, and 6 are all factors of 12.)
4×3 and 6×2

Inverse - Opposite in effect; reversed.
Example: $3 + 2 = 5$ in addition
$5 - 2 = 3$ in subtraction
Addition and subtraction are inverse operations.
Multiplication and division are inverse of one another.

Odd Number - Whole numbers that are not evenly divisible by 2.
Example: 1, 3, 5, 7...

Ordinal Number - A number used to tell "which one" in sequence.
Example: first, second, third, etc.

Permutation - A permutation is any one linear arrangement of the total number of possible linear arrangements of a given set of objects.

Prime Numbers - A whole number greater than 1 that has only itself and one as its factors.
Example: 2 and 13 are prime numbers.
$2 \times 1 = 2$ $13 \times 1 = 13$

Product - Result or answer in a multiplication operation.

Rule for Substitution - Two or more like symbols in an open sentence or equation require the same number to be substituted each time the symbol is repeated. Thus:

$\square + \square = 10$

$5 + 5$ makes a correct substitution. While $2 + 8 = 10$ would make a true statement, it breaks the rule for substitution.

$\square \times \square \times \square = 8$
$2 \times 2 \times 2 = 8$

When two unlike symbols are used in an equation, the numbers may be different. Thus:

$\square \times \triangle = 15$

3×5
5×3
15×1
1×15

All may be correct answers to the equation.

Set - A set is a collection of things or objects.
Example A: The odd numbers that are greater than 5 and less than 15. (7, 9, 11, 13)
Example B: The coins in our money that have a value of less than one dollar. (cent, nickel, dime, quarter, half-dollar)

Sum - Result or answer in an addition operation.

SOME BASIC LAWS OF MATHEMATICS

Commutative Law for Addition - The order of adding numbers does not affect their sum; or briefly, reverses in addition have equal sums.

$$\square + \triangle = \triangle + \square \qquad 2/3 + 3 = 3 + 2/3$$

Example: $3 + 2 = 2 + 3$

Commutative Law for Multiplication - The order of multiplying numbers does not affect their product, or reverses in multiplication have equal products.

$$\square \times \triangle = \triangle \times \square \qquad 2/3 \times 3 = 3 \times 2/3$$

Example: $4 \times 3 = 3 \times 4$

Law for 1 - When a number is multiplied by 1, the product is the same as the number.

$$\square \times 1 = \square \qquad 5/8 \times 1 = 5/8$$

Example: $4 \times 1 = 4$

Addition Law for Zero - When we add zero to a number, the sum is the same as the number.

$$\square + 0 = \square$$

Example: $2 + 0 = 2$

Multiplication Law for Zero - Any number multiplied by zero produces a product that is zero.

$$\square \times 0 = 0$$

Example: $3 \times 0 = 0$

Distributive Law - Term applied to distribution.

$$\square \times (\triangle + \nabla) = (\square \times \triangle) + (\square \times \nabla)$$

Examples: $3 \times (20 + 4) = (3 \times 20) + (3 \times 4)$

$3 \times 24 = 60 + 12$

$72 = 72$

Associative Law for Addition - Regrouping three or more addends does not change the sum.

$$\square + (\nabla + \triangle) = (\square + \nabla) + \triangle$$

Example: $3 + (4 + 6) = (3 + 4) + 6$

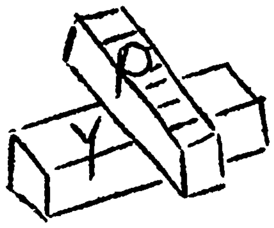
Associative Law for Multiplication - Regrouping three or more numbers (factors) being multiplied does not affect the product.

$$\square \times (\triangle \times \nabla) = (\square \times \triangle) \times \nabla$$

Example: $2 \times (3 \times 4) = (2 \times 3) \times 4$

SYMBOLS*

CROSS

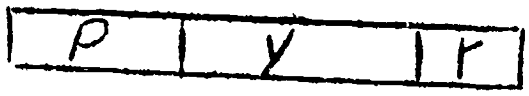


purple-yellow cross

TOWER

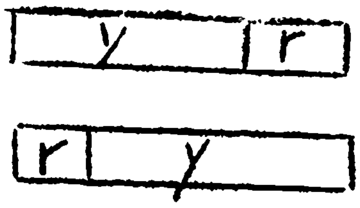
red-purple-yellow tower

TRAIN



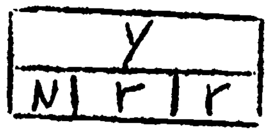
purple-yellow-red train

END-TO-END, ADD



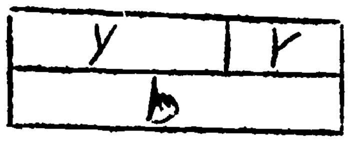
yellow, red rods end-to-end, touching "yellow plus red" or "red plus yellow"

SIDE-BY-SIDE DIVIDE (with remainder)



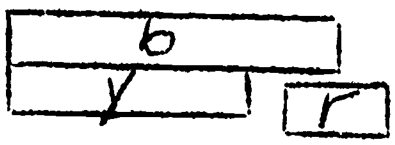
yellow, red rods side-by-side, touching, "How many 2's in 5?" "How many left over?"

FITS, MATCHES AS LONG AS



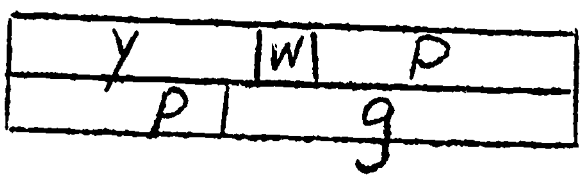
"yellow, add red fits black" or "yellow and red make black" or "yellow and red are as long as black", or 5 + 2 =7

MINUS



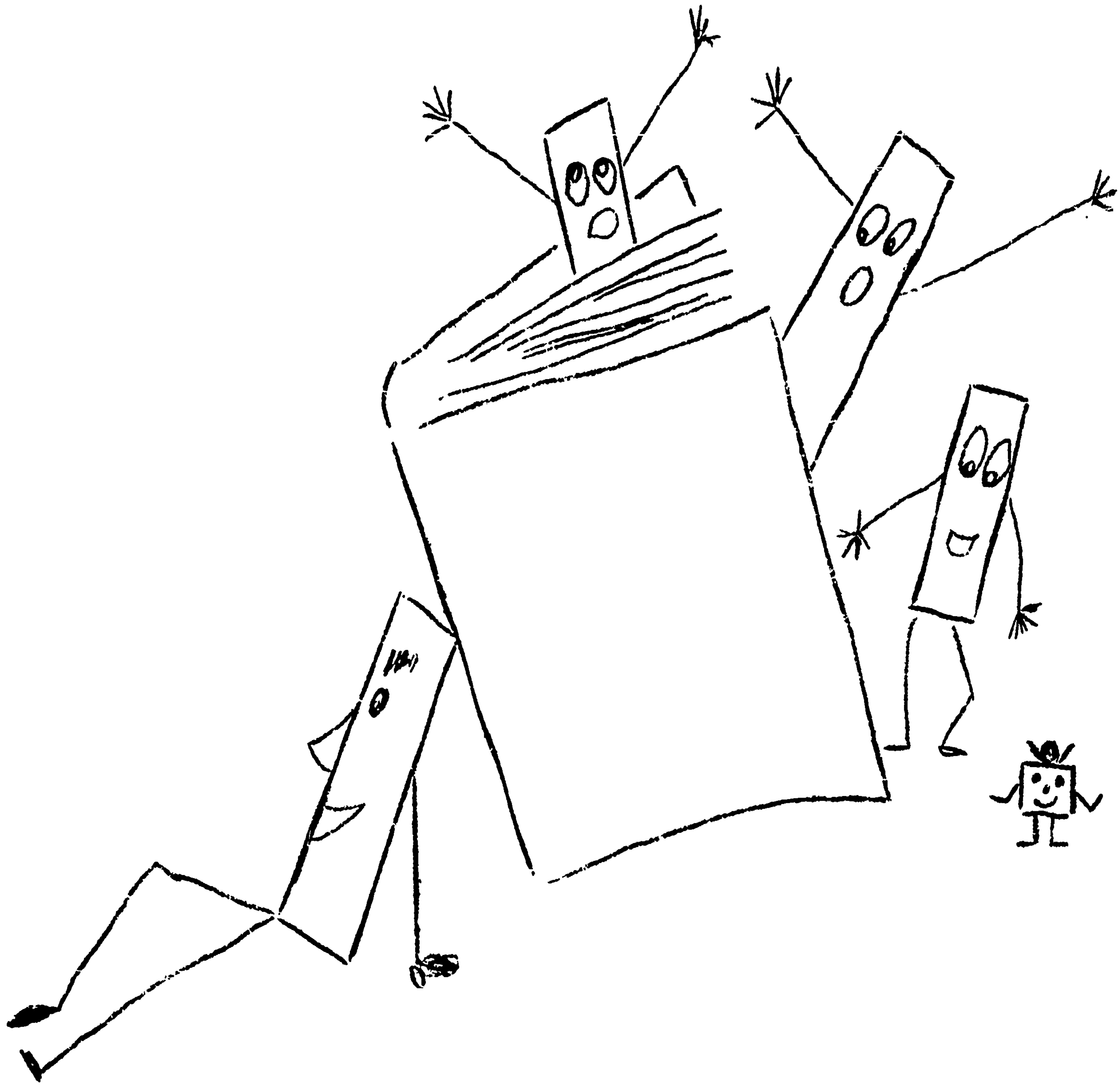
"Black subtract yellow equals red" or put yellow on top of black for easier viewing of remainder.

EQUIVALENT



Two equivalent trains are of equal length.

* B-2, pp. 7-22



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B-2. Mathematical Awareness, Part I, John V. Trivett, 1962.

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B-5. An Introduction to the Cuisenaire Rods, Reprinted from Shady Hill News, Shady Hill School, Cambridge, Massachusetts.

B-6. Elementary Rhythms with the Cuisenaire Rods, Book I, 1963.

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B-7. Modern Mathematics With Numbers in Colour, C. Gattegno, 1961.

B-8. Numbers in Colour, A New Method of Teaching Arithmetic in Primary Schools, G. Cuisenaire and C. Gattegno, 1959.

* Any book on this list may be obtained from the Cuisenaire Company of America, 235 East 50th Street, New York 22, New York. All Cuisenaire materials are NDEA approved.

** These books are for the pupils use but this committee recommends them for teacher reference.